

## Volvelles of knowledge. Origin and development of an instrument of scientific imagination (13<sup>th</sup>-17<sup>th</sup> centuries)

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### ABSTRACT

This contribution reconstructs the history of volvelle in movable books; books created for a wide range of different purposes (teaching, mnemonics, play, divining, etc.) including mechanical or paratextual devices demanding or soliciting the interaction of the reader. The investigation runs from hand-written books, considering some specific editorial genres, including calendars, books of fate, anatomical flap books, navigation handbooks etc.

### KEYWORDS

Movable books; Ramon Lull; Matthew Paris; Regiomontanus; Apianus; Leonhard Thurneysser; Ottavio Pisani; Book of fate; Volvelle; Anatomical flap books.

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It is well known that from the second half of the 16<sup>th</sup> century, the practical and direct observation of phenomena and objects of nature and their manipulation marked a radical change in the methods of scientific investigation and the understanding of the world and its material complexity. In this changing and composite context of economic, social and cultural scenarios, books, and above all illustrated scientific books, became the medium of new knowledge, the essential support for understanding the operation of the universe, in which man had conquered an absolute centre-stage role. As acutely stated by paleographer and book historian Armando Petrucci, what could only be imagined by means of the words of a written discourse began to be known through the vision offered by printed images (Petrucci 1988, 148). The rhetoric and discursive power of the image was thus used to “illustrate” the scientific revolution, enshrining, in the European culture of the late 16<sup>th</sup> and 17<sup>th</sup> centuries, the significant success of illustrated scientific books, and more particularly, that of “movable books”. This expression refers to books created for a wide range of different purposes (teaching, mnemonics, play, divining, etc.) including mechanical or paratextual devices demanding or soliciting the interaction of the reader.<sup>1</sup> As in the case of *flaps* (designed and used to cover and then reveal one or more underlying images, used above all in anatomical books), or *volvelles* (rotating paper or membrane discs, shaped and overlapping and fixed to the page with one or more pins, allowing each disc to be independently rotated around its axis). The interactivity justifying the generic yet effective name of *movable books* for this type of published products, is obtained mainly through: the movement by the reader of some elements inserted in the pages; the sequential arrangement of images held together by tabs opening the book in a bellows-fashion; the rapid and sequential scrolling of the pages, animating the represented figures to create the illusion of movement; the decomposition of the support, adding a three-dimensional effect to the portrayed scenes.

For the readers, called on to operate or manipulate the mechanical device or system incorporated into the paper or parchment pages of the book, the material action became a physical, multi-sensory as well as an intellectual experience; in this way, the manual gesture transformed the mechanical or paratextual device created by the author into a semiotic and communicative space enhancing the semantic value of the text and generating original and unexpected iconic reading spaces. Before the eyes and in the hands of the reader, the book enhanced all its intended uses, becoming a physical space for self-learning, placing this world and the world beyond the book in direct relationship, a medium of knowledge and the instrument for experimenting that knowledge.

Volvelles and flaps were the two most commonly used devices in scientific books between the 16<sup>th</sup> and 17<sup>th</sup> centuries, but in fact their origins lay further back in history, in the manuscript culture; the most significant examples can be found in the works of the English Benedictine monk Matthew Paris (1200 ca.-1259), who we will investigate further on, and the Catalan-speaking Majorca-born philosopher Ramon Lull (1232/33-1315).<sup>2</sup> And yet, to understand the influence of the Lullian

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<sup>1</sup> On “movable books”, in addition to the now classic article by Sten G. Lindberg (Lindberg 1979), refer also to the monograph by Suzanne Karr Schmidt (Karr Schmidt 2018), the most complete study on some types of “movable” books and prints from the Renaissance period, above all from Germany and Northern Europe. See also Franchi 1998, Sarlatto 2016, Reid-Walsh 2018. Finally, I refer to my own contribution, Crupi 2016.

<sup>2</sup> A large and diversified bibliography exists on the complex intellectual figure of Lull, bearing witness to his original and indeed complex critical fortune. One useful tool for historical and bibliographical guidance on the works of Lull is offered by the website [Qui és Ramon Lull?](#), curated by the Centre de Documentació Ramon Lull de la Universitat de Barcelona.

philosophical system for several centuries in some very different fields and applications of knowledge, we need to look briefly at some of its peculiarities, above all in relation to the history of the volvelle. Concentrating a huge amount of information in this tiny device, Lull developed a sophisticated logic mechanism, based on the combinatorial technique and aiming to obtain the truth, or rather, provide the key for elaborating the truth or falsity of a proposition.

### Ramon Lull's *Ars*

In its original meaning, the *Ars* (the title he gave to his method) was meant to provide an incontrovertible and rational instrument for converting the infidels, starting from the conceptual baggage of Aristotelian metaphysics and logic and by some principles shared by the three monotheistic religions. Principles traceable to features or attributes of God, defined as “dignities”, the foundation on which Lull laid his *ars* (Bonitas, Magnitudo, Eternitas, Potestas, Sapientia, Voluntas, Virtus, Veritas, Gloria) and the universality of which was such as to be shared by men of Christian, Hebrew or Muslim faith. Indeed, in the Iberian Peninsula, and particularly in Majorca, Christianity was in direct contact with Hebrew and Muslim mysticism, and remained so for a long time, even after the Jews and Moors were ousted from Spain between 1492 and 1505.

In its most advanced stage of development (1290-1308),<sup>3</sup> Lull's *Ars* had four figures and an alphabet of nine letters from B to K, representing the series of nine dignities. In particular, in the fourth figure (Fig. 1) the sequence of letters is portrayed and replicated on three volvelles, three concentric and overlapping rotating discs of decreasing size, thus obtaining all their possible combinations.



Figure 1. Ramon Lull, Fourth Figure in *Ars magna, generalis et ultima* (Lyon 1517)

See also the studies by: Pring-Mill 1961, Platzeck 1962-1964, Yates 1982, Rossi 1983, Bonner 2007, Eco 2007, Fidora & Rubio 2008, Eco 2016.

<sup>3</sup> Anthony Bonner (Bonner 1985) characterised the *Ars* into three chronological phases, starting from the previous studies by Tomàs and Joaquim Carreras Artau, Frances Yates and Robert Pring-Mill.

Certainly, the most renowned and most successful in Lullian tradition, as it configures «una combinatoria universale e illimitata» (Eco 2007, 351), this figure is the one out of the four with the highest degree of symbolic and formal abstraction. The logic formalisms, paper engineering and (monochrome and polychrome) graphic devices developed by Lull thus bring life to a combinatorial mechanism aiming to mathematicise the processes of knowledge. To know God and his creation (along the whole scale of Creation) Lull in fact developed a rational system based on a finite number of concepts: a “conceptual atomism”,<sup>4</sup> which in the theory of the elements found «un fondamento universale, religioso e scientifico, per un’arte infallibile, tale da poter funzionare a tutti i livelli del creato» (Yates 1982, 15). So, the concept of dignities led Lull to formulate the idea of a universal science, which hypothesised that every branch of knowledge consisted in a finite number of fundamental categories or principles and that, exploring all their possible combinations, man would be able to understand and govern the principles of all particular sciences. In the Lullist tradition, this idea which was to become «an intellectual obsession in European culture» (Rossi 2000, 29; cited by Koetsier 2016, 74), manifested itself in the aspiration of finding a *clavis universalis* offering access to the whole encyclopaedia of knowledge, «riprendendo le suggestioni delle moltissime enciclopedie medievali e anticipando l’utopia enciclopedica della cultura rinascimentale e barocca» (Eco 2007, 366).

Lull’s philosophical thought referred to classic, patristic and scholastic traditions, and to the Platonic or Neo-platonic tradition in particular. Yet, as explained, he was also familiar with the mysticism, science and philosophy of both the Hebrew and the sophisticated Spanish Muslim civilisation. Indeed, critics specifically underlined the analogies between the Lullian functional system and a divinatory device (the *Zâ’irajah*), used in ascetic circles of Sufism, a predictive device using the principle of concentric circles to forecast future happenings based on complex combinatorial rules.<sup>5</sup> More specifically, the *Sefer Yetzirah* or *Book of Creation* was identified as one of Lull’s probable sources; this is the text of reference for the whole Kabbalistic tradition (Hayman 2004), as well as, in turn, of the *Zohar*, the foundational work in the literature of Jewish mystical thought.<sup>6</sup> The visual, as well as logic and formal, association of Lull’s fourth figure with the wheel described in the *Sefer Yetzirah* is inevitable; here the twenty two letters of the Jewish alphabet are arranged, which according to Kabbalist tradition contain the name or names of God, and which through the combinatorial movement of the wheel generate an infinite number of permutations (Eco 2007, 366). It was precisely these points of contact or analogy, the *ars* being a Kabbalist type method, without recourse to the

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<sup>4</sup> The definition of “conceptual atomism” was coined by the Dutch scholar Teun Koetsier (Koetsier 2016, 56).

<sup>5</sup> «Another technical rule for alleged discovery of the supernatural is the *zâ’irajah* which is called “*Zâ’irajah* of the world”. [...]The form of the *zâ’irajah* they use is a large circle that encloses other concentric circles for the spheres, the elements, the created things, the *spiritualia*, as well as other types of beings and sciences. Each circle is divided into sections, the areas of which represent the signs of the zodiac, or the elements, or other things. The lines dividing each section run to the center. They are called chords. Along each chord there are sets of letters that have a conventional (numerical value)» (Ibn Khaldun [1958]). See Link 2010 as well as Urvoy 1980 and 1990, Vega 2003.

<sup>6</sup> «It was in medieval Spain that the Kabbalah reached its highest development, and that acme coincides with the appearance of Lullism. The *Zohar* was written in Spain in 1275 circa. In 1274 on Mount Randa Lull had the vision which revealed to him the two key figures of the art. There are many points of contact and similarities between Kabbalism and Lullism» (Yates 1982, 17). Yates also defined the *ars* as a «forma medievale della kabbalah cristiana» (ivi, 18). See also Millás Vallicrosa, 1958.

linguistic esotericism of the Jewish alphabet, which led to Lull's controversial fortune. Assuming the coloured triangles and rotating circles of the *Ars inveniendi veritatem* as the essential purpose of his works, for at least two centuries the foundations of his doctrine were misunderstood, indeed attributing to him clearly apocryphal ideas and works.

It is however undeniable that the convergence of paper engineering, rotational movement and combinatorial method broadened the scope of application of Lull's *ars*, touching sectors of scientific knowledge which mathematicised not only some heuristic processes but also aspects of the understanding of nature (for example medical astrology), which interacted with everyday human life. Having found the artifice, it could be used, and indeed was used, in a multitude of applications (from cryptography to rhetoric, from medicine to astrology), coming together with the astronomical tradition and the tradition of divinatory texts, which used *rotulae* or volvelles to measure and estimate the latitude of a point on the earth's surface and the position of the stars, or to calculate, for example, the date of moveable feasts such as Easter.

## Matthew Paris' Paper Engineering Expedients

And here we must return to Matthew Paris,<sup>7</sup> who was certainly one of the first to include moving elements (not only volvelles) in a handwritten book. He accompanied his *Chronica majora*,<sup>8</sup> a considerable historical narration composed in the mid-13<sup>th</sup> century, with an original series of maps showing the paths and routes that, through major European cities, linked London to the most important destinations of religious pilgrimage, Rome and Jerusalem. These maps, which offered a geographic context of the historical narration for the benefit of the cloistered brothers, had folded parchment flaps to the top and sides, which opened and closed could modify the itinerary and the represented geographic space.<sup>9</sup> The extent of the natural confines of the manuscript, functional to the representation of a sacred geography, thus became a multi-sensory experience for the reader, as in their eyes it offered the possibility to undertake an interior journey of meditation, a mental pilgrimage that could be remodulated, open to alternative itineraries (Crupi 2016, 28).

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<sup>7</sup> Throughout his monastic life, M. Paris worked from St. Albans Abbey, in the English county of Hertfordshire. «Although his surname, which he usually wrote as "Parisiensis" or on rare occasions "de Parisus," could suggest French origins, nothing in his work or elsewhere indicates that he was anything other than an Englishman trained in a Benedictine monastery to be characteristically literate in both Latin and Anglo-Norman French» (Lewis 1987, 3). On M. Paris, see Vaughan 1958, Lewis 1987, Connolly 2009.

<sup>8</sup> «It was not, however, a single-handed undertaking. Up to the annal for July 1235, it consists of a revision of the *Flores Historiarum* written by Roger Wendover and represents Matthew's work only in the continuation of the monumental history to 1259. An illustrated autograph copy of the *Chronica Majora* survives in three volumes: MS 26 in Corpus Christi College, Cambridge, containing the annals from Creation to 1188; MS 16, also at Corpus Christi College, with annals from 1189 to 1253; and British Library MS Roy. 14. C. VII, which contains the entries from 1254 to Matthew's death in 1259 on fols. 157 to 218. The relative scale of Paris's contribution to the *Chronica Majora* may be suggested by noting that his annals for the last twenty-four years (1235-1259) are roughly as long as the whole preceding history from Creation» (Lewis 1987, 9).

<sup>9</sup> Matthew Paris's Map of the Route to Jerusalem. St Albans, c.1250, British Library Royal Ms. 14 C vii, f. [http://www.bl.uk/onlinegallery/sacredtexts/mparis\\_lg.html](http://www.bl.uk/onlinegallery/sacredtexts/mparis_lg.html).

Ms. 26 of Corpus Christi College in Cambridge, which hosts the first volume of the *Chronica*, also has a volvelle,<sup>10</sup> here used to optimise the *computus* (Borst 1997; Capasso and Piccari 2000), *i.e.* the method used to determine the date of Easter, based on a lunar cycle of nineteen solar years. The *computus* was traditionally composed of mostly circular diagrams, which however due to their staticity forced the reader to rotate the codex, or their own body around the codex, in order to read the numerical correspondence between solar and lunar year, thus determining the dates of the liturgical celebrations. The innovation introduced by Matthew Paris as Daniel Connolly well observed allowed readers to rotate the disk, aligning it to their bodies thus facilitating the reading of the graphic indications and the data they contained (Connolly 2009, 65). The ingenuity of the mechanical artifices introduced by Matthew Paris in his *Chronica*, which in both cases required both a physical and a performing action, transformed the reader's manual gesture into an intellectual experience of knowledge, thus enabling them to become not merely the interpreter of the text but also its architect, whose intervention enables the algorithm that generates meaning.

## Wheels of Fortune and “Books of fate”

These new technologies of book production were also used in another work by the ingenious and eclectic English monk, who around 1250 copied and illustrated a collection of seven works on the fate of a codex, the Ms. Ashmole 304, better known as the *Book of Fate* and kept at the Bodleian Library in Oxford.

«I libri di sorte sono testi che presentano una serie di domande e risposte sul destino dell'uomo. Le domande sono scritte in apertura del libro, spesso all'interno di cartigli intorno alla ruota della Fortuna, e si raggiunge l'oracolo finale per tappe successive, dopo aver compiuto più volte un'azione (ad esempio lanciare i dadi) che permette di ottenere un numero e passare alla fase successiva del gioco. Spesso il procedere è scandito da un'*escalation* dall'umano al divino che vede come interlocutori del giocatore soggetti diversi (animali, fiumi, re, apostoli, divinità, ecc.), che vengono anche raffigurati nelle pagine con disegni o incisioni» (Urbini 2019, 129–130).

Where present, the volvelles were usually activated by the verso of the leaf or front cover, thus being invisible to the reader until the book was opened, the divinatory position of the pointer indicated the calculation of the score required to reach the answer.

Although Matthew Paris's codex has no trace of volvelles, a series of convincing philological evidence,<sup>11</sup> based on the comparison with some copies of the manuscript,<sup>12</sup> allow us to suppose that it contained at least three and that one of these was very similar to that represented in Bernardus Silvestris's *Experimentarius* preserved in one of the copies, the Ms. Digby 46.<sup>13</sup> The volvelle consists

<sup>10</sup> Cambridge, Corpus Christi College, Ms. 26: Matthew Paris OSB, *Chronica maiora* I, f. Vr.

<https://parker.stanford.edu/parker/catalog/xf352tc5448>.

<sup>11</sup> In particular, Allegra Iafra's studies are worthy of mention, also providing an excellent summary of the ecdotic and structural aspects of the codex: Iafra 2011, 2012, 2013.

<sup>12</sup> The reference copies of the Ashmole 304 codex are above all the manuscripts Cambridge, Magdalene College, Pepys 911 (also dating back to the 13th century) and the Oxford, Bodleian Library, Digby 46 (14<sup>th</sup> century).

<sup>13</sup> See Digital Bodleian, *Various works including Liber fortunae, also known as Experimentarius*.

<https://digital.bodleian.ox.ac.uk/inquire/p/c8b55e2b-24f3-41f6-bd15-046e8a657b6f>.

of two wooden toothed wheels of different sizes, positioned in such a way that the rotation of one causes the movement of the other. The parchment disc stuck to the larger wheel has three numbered concentric circumferences:

«Each of them corresponds to a series: 1-28, 1-10, 1-12. This mechanism allows to use any of the works in the manuscript, since the random number one needs cannot exceed 28, 12 or 10 (except for the *Divinacio Ciceronalis*, numbered 1-20). In the centre of the circle there is a figure pointing at the number. Probably, the idea was this: the *quaerens* should lightly put his finger on the numbered strip he needed to use, in correspondence to the pointing finger of the figure. Then, without looking down he should spin the small cogwheel to activate the big one. After few rounds, he should simply check which number of the series had ended up below his finger» (Iafrate 2012, 58).

Usually composed of an introduction, sometimes including instructions for use, and lists of questions and answers, books of fate were very common from the Middle Ages despite their formal condemnation by the Church, which forbid their use. We may mention some marvellous examples: *Il Libro delle Sorti* (1482) by Lorenzo Spirito Gualtieri (1426-1496),<sup>14</sup> which enjoyed considerable fame until the 17<sup>th</sup> century;<sup>15</sup> *Il Trionfo di Fortuna* (1527) by the Ferrara-born Sigismondo Fanti, who lived in the late 15<sup>th</sup> and early 16<sup>th</sup> centuries (Fanti 1527), accompanied by precious woodcut tables and enriched by a precise illustration of the rules the players had to follow in order to obtain the divinatory answers; *Le sorti intitolate giardino di pensieri* (1540) by Francesco Marcolini (1500 circa-1559 circa),<sup>16</sup> accompanied by over two thousand rhyming tercets by Venetian scholar Ludovico Dolce (1508-1568), which proclaimed the outcome of each of the as many entertaining oracular combinations, which however, compared to the works of Spirito and Fanti, used cards and not dice. The first printed book of fate with a volvelle is the incunable *Losbuch*, printed in Basel in 1485 by Martin Flach<sup>17</sup> and published three years later by Lorenzo Spirito, in which «il gioco è sovrinteso da un mostro marino che decora la ruota e da cinquantadue animali parlanti che danno il responso» (Urbini 2019, 136). And thus, rituals based on the imponderability of fate found their way into the iconographic, textual and performing framework offered by this very particular graphic and literary genre, represented by books of fate: books in which the boundary between the sacred and the profane, between pleasure and moral edification, between play and knowledge, was undefinable; books which were combinatorial machines, both prophetic and entertaining, able to playfully imitate the way in which fate works over the course of human events; books whose divinatory function, supported by rich and sometime complex iconographic devices, was activated by instruments based on imponderability and fate, such as dice, cards, coins, or, precisely, volvelles. They had the ability to exercise great suggestion in the visual imagination of the “players”, as they evoked, in the popular iconographic form of the *rota Fortunae*, the capricious nature of fate and its impenetrable blind will (Helfand 2002). But in this

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<sup>14</sup> From the same year as the subscriptio of the handwritten codex (1482), kept at the Marciana National Library in Venice (Ms. IX.87=6226), we find the editio princeps, published in Perugia by Stephanus Arndes, Paulus Mechter, Gerardus Thomae, of which only one copy is still known to exist, kept at the Stadtbibliothek in Ulm. See Urbini 2006, to whom we owe the discovery and study of the handwritten codex.

<sup>15</sup> See Zollinger 2007.

<sup>16</sup> See Marcolini 1540. A second edition followed this, completing the figurative corpus and rethinking the self-censorship of the text (Marcolini 1550).

<sup>17</sup> The only copy known today is kept at the Staatsbibliothek in Berlin and is now available in digital format.

case with a substantial difference: in rotating the volvelle the player became responsible for his own fate, assuming the role of Fortune itself, both the object and subject of its imponderability. This the book became a space interpretative, and the volvelle (like dice or cards) the device transforming the book into a material contraption for experimenting time, the future and the whims of Fate.

## Ciphered languages

Let us return to Lull, because according to US historian David Kahn, Lull's *ars inveniendi veritatem*, expressed in the computational mechanics of overlapping rotating disks, was the basis for the creation of language ciphering systems and, thanks to his example, the invention of a ciphering volvelle by Leon Battista Alberti (1404-1472). In his work *De cifris*, composed around 1466 for the apostolic secretary Leonardo Dati, to make his correspondence more secure, Alberti created a polyalphabetic cipher based on the sliding of two concentric disks (Fig. 2) «contenenti un alfabeto ordinato per il testo in chiaro (testo da cifrare) e un alfabeto disordinato per il testo cifrato (testo risultante)» (“Disco cifrante” 2018).



Figure 2. L.B. Alberti, Ciphering disk in *A Treatise on Ciphers* (Torino 1997)

In truth, the expedient had already been the focus of interest of the Paduan scientist and humanist Giovanni Fontana (1390 ca.-1454 ca.), who in his *Secretum de thesauro experimentorum ymaginationis hominum*, dated to around 1430, had proposed «a series of contraptions and machines for memory, composed of a fixed part (wheels, spirals, cylinders) and a mobile, variable part used to change the combinations of signs in the system» (Muccillo 1997). This artifice, in the form of ciphered wheels, was also used by the German abbot Johannes von Heidenberg (1462-1516), better known by the

humanist name of Trithemius, in his work on *Steganography*, published posthumously in 1606 but immediately condemned to the *Indice* (1609) accused of being a work of occult astrology.<sup>18</sup> In truth, Trithemius developed complex numerical and monoalphabetic substitution ciphers using volvelles. In monoalphabetic ciphers, the calculating mechanism of the volvelle allowed the place of every letter in the message to be taken from the letter found at a distance of  $x$  in the ordinary alphabet. As Umberto Eco explained: «Non importa quanto Tritemio sia stato ispirato da Lullo, perché in ogni caso l'influenza sarebbe stata puramente di ordine "grafico": per Tritemio le rotule non servono a produrre argomenti, bensì a cifrare e a decifrare [... perché lo] steganografo non è interessato al contenuto (e quindi alla verità) delle combinazioni che produce» (Eco 2007, 376). His combinatorial system is purely formal, based on the permutation of one system of symbols with another. And the Neapolitan philosopher and scholar Giovambattista Della Porta (1535-1615) was fully aware of this; in *De furtivis literarum notis* (1563) (Fig. 3) and in its later extension, *De occultis literarum notis* (1593), summarised the cryptographic systems known until that time, accompanying his lucky work with tables, illustrations and moving cipher discs, which allowed the reader not only to understand but above all the appreciate and experiment the operation of the mechanical device.

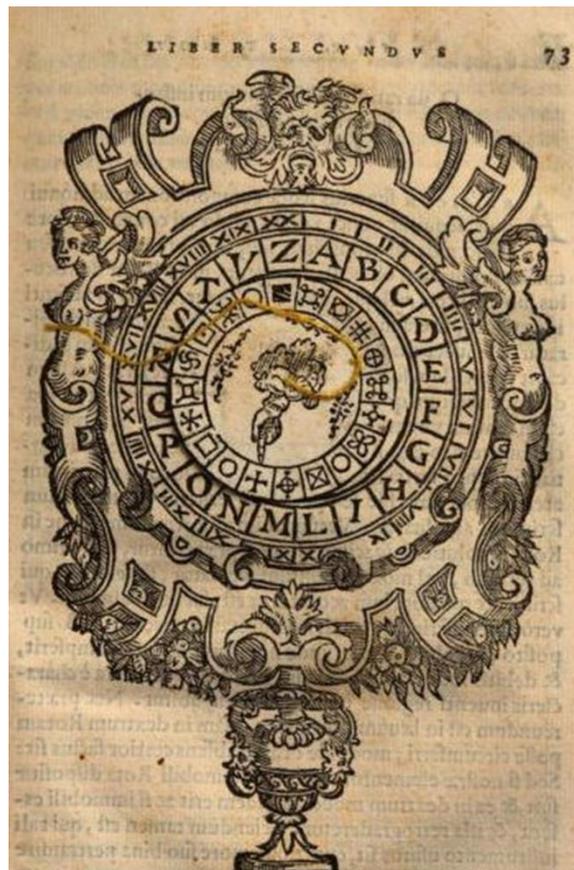


Figure 3. G.B. Della Porta, *De furtivis literarum notis* (Napoli 1563)

<sup>18</sup> In contrast the specifically cryptographic techniques, steganography is not based on the transformation of the text but rather the physical hiding of the message. See also Trithemius 1518.

## ***Ars memoriae and Rhetoric***

In the same period, Giordano Bruno (1548-1600), in *De umbris idearum* (Bruno 1582), sought «un punto di convergenza tra l'immaginario attivo del lullismo (da cui desume proprio la strumentazione enciclopedica delle ruote alfabetiche) e le tradizionali procedure delle mnemotecniche retoriche» (Bondi and Torre 2019, 84). Although the device designed by Bruno was not actually movable in its printed version, it was potentially. It was in fact made of a variable system of concentric wheels, the circumferences of which were divided into thirty cells marked by Latin, Greek and Jewish letters, each of which corresponded to an iconic symbol. Once assembled, the movement of the wheels offered the association of word strings (*loci memoriae*) with what one wanted to bring to memory (*imagines agentes*). The reader therefore had the task of activating and combining the various diagrams and use them as supports for memory and imagination. «Se, ad esempio, ai *loci* della prima ruota associamo nomi di personaggi del mito, a quelli della seconda le azioni che essi compiono, e a quelli della terza gli attributi che li caratterizzano, la lettura mentale della sequenza di *loci* posti lungo l'asse di ogni raggio (AAA, BBB, etc.) può restituirmi la sintesi di una vicenda mitologica o, più semplicemente, una frase di senso compiuto» (Bondi and Torre 2019, 85). A couple of years after the publication of Bruno's text, the Cypriot Giason Denores (1530-1590 circa) printed his work on rhetoric (Denores 1584), at the end of which were three moving "wheels". Volvelles were used to put into practice «anchora con molto maggior brevità, & facilità» the three genres of eloquence: epideictic, deliberative and judicial. Denores, who in some way owed much to Lull's combinatorial model, had developed a system by which the reader could learn to find suitable arguments according to the oratory genres, contexts and occasions:

«in un batter d'occhio troveremo le prove particolari in ciascuna questione [...] e conseguiremo non soltanto l'insieme degli argomenti e delle prove oratorie, ma anchora in gran parte la loro disposizione e ridurremo il nostro intelletto a pochi capi senza tornar a leggere ogni volta che vorremo componer una Orazione i Libri della Rhetorica» (Denores 1584, 264r).

## **From Astronomy to Astrology. Volvelles for measuring time**

The fortune of the volvelles and their functional multi-purpose made their mark along with the revolution of movable type printing, which extended their use (limited earlier practically exclusively to the holder of the codex and its few lucky readers) to a wider audience able to exploit their interactive features. A revolution that, for this and other movable devices led to the research and invention of technical solutions which, in terms of typographic composition and publishing, had to reconcile mass production with the creation of unique pieces, which needed to be wholly or partially assembled by hand by the reader or, in a subsequent post-production phase, by the typographer and the bookbinder. In particular, the presence of paper devices with moving parts became an integral part of woodcut prints and "movable books", above all of a "scientific" nature; indeed, for over a century the volvelles became a popular and indispensable complement to books on the art of

navigation and astronomy manuals, an educational support able to transmit technical information in an interactive format.<sup>19</sup>

An emblematic testimonial is offered by the *Kalendario* (1476) by the German mathematician Johann Müller, better known by the humanist name of Regiomontanus (1436-1476).<sup>20</sup> An innovative work, first of all – as is known – because it the first known Italian title page which, in the coded form of a caudate sonnet, contains not only the title and date of the publication but also the name of the author and that of the printers (Fig. 4).<sup>21</sup>

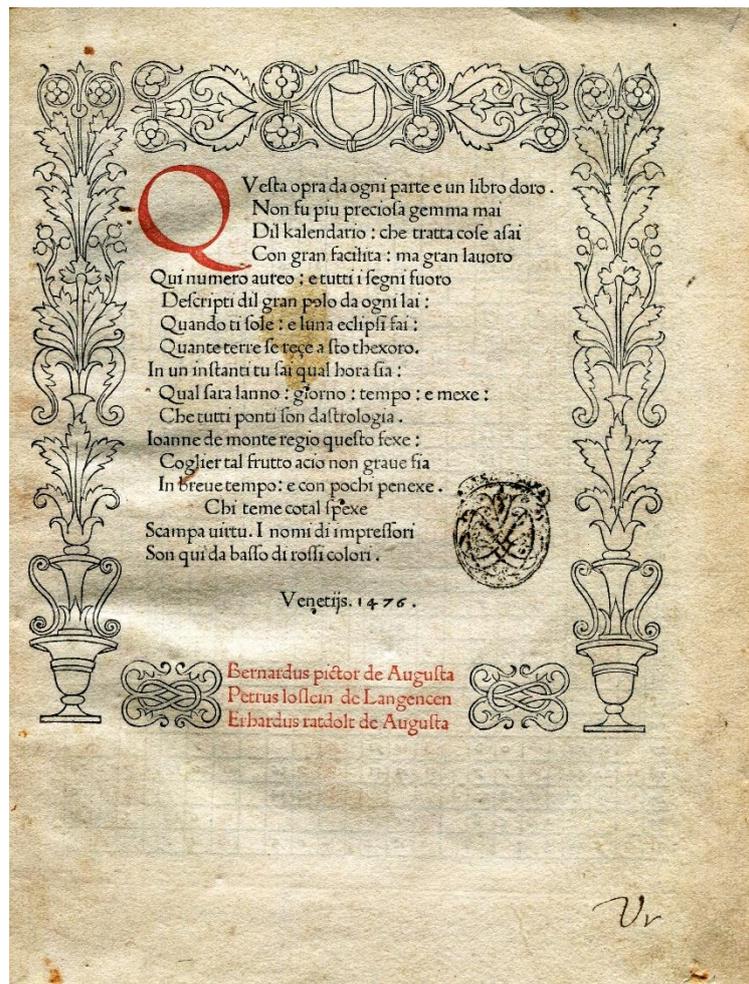


Figure 4. Regiomontanus, *Kalendario* (Venezia 1476). Courtesy Biblioteca Universitaria Alessandrina – Roma

<sup>19</sup> On movable devices in astronomy books, see Gingerich 1993 and also Bennett 2011, Kremer 2011.

<sup>20</sup> See Zimmer 1990, O'Connor and Robertson 2004.

<sup>21</sup> The 1476 *Kalendario*, the front cover of which is framed by a woodcut decoration, was published at the same time in both Italian vernacular and Latin; the latter is composed in hexameters which, in substance, reproduce the contents of the sonnet in the vernacular. The 1476 edition was in turn preceded by two versions, in Latin in Roman characters and in German with Gothic characters, both without title or date (although with all probability dated to 1474) and both printed in Regiomontanus's printing house.

Moreover, it is renowned in the history of printed books as the first printed work which includes several movable devices, which, specifically, are used to mathematically calculate time: *Lo instrumento de le hore inequale*, consisting of a thread fixed to the centre of a graduated dial and used as a cursor; *Lo instrumento del vero moto de la luna*, represented by two overlapping volvelles, which rotate on a fixed disc drawn on the page (Fig. 5);



Figure 5. Regiomontanus, *Kalendario* (Venezia 1476). Courtesy Biblioteca Franceseana - Milano

*El Quadrante del Horologio Horizontale* and *El Quadrato Generale de le hore*, both fitted with moving brass pointers, fixed to the page with a rivet. Regiomontanus provides precise instructions to the reader on how to use the volvelles, which he calls either “*rotule mobile*” or “*rotelle*”, «le qual respondeno a li doi numeri lunari posti nel kalendario» and which are needed to determine the position of the Moon in the sky throughout the year. They rotate around a fixed circular dial, consisting of graduated concentric rings and the representation of the constellations of the Zodiac. A thread fixed to the central axis of the dial is used as a hand, which the author invites the read to fix with a dot of wax:

«Et doue quello numero finira mette li el filo del instrumento: & uolta la rotella magiore: per infin chel capo de quella cioe la croce giasa sotto el filo. & cossi stando la rotella: fermela con cera de dentro uia. perche el sito di quella sara tale per tutto lanno» (*Del loco vero de la luna*).

Other images and tables in the *Kalendario* are on the other hand functional to forecasting solar and lunar eclipses from 1475 to 1530, and calculating the days of the year on which some moveable feasts like Easter.

The introduction of volvelles as educational supports for transmitting «technical information in an interactive format» (Gravelle, Mustapha and Leroux 2012) is even more exemplary in *Cosmographicus liber* (Apianus 1524)<sup>22</sup> by the German mathematician and cosmographer Petrus Apianus<sup>23</sup> which, as science historians well know, was a significant work for 16<sup>th</sup> century astronomical, geographical and cartographic studies, even though its editorial fortune in fact began from the second (1529) and above all the third edition (1533), both curated by the Dutch mathematician Gemma Frisius.<sup>24</sup> Cosmography, which was to all extents and purposes a branch of the mathematical arts, was understood as the intersection of geography and astronomy: it represented the astronomical foundations of the description of the Earth, with significant practical consequences for topographic surveying, cartography, the art of navigation and the creation of surveying, measuring and observation instruments. Apianus exploited the notions of the Ptolemaic astronomical culture, described above all in the canonical text on spherical astronomy, *De sphaera mundi* by Johannes de Sacrobosco,<sup>25</sup> and divulged by some European universities, among which, within the renaissance of mathematical and astronomical studies, the University of Vienna,<sup>26</sup> where the study of astronomy was closely linked to that of medical astrology. And yet, also beyond the universities there was a proliferation of instrumental culture and independent publishing of “mathematical” works, inaugurated by the *Elucidatio fabricae ususque astrolabii* (1513) in which its author, Johannes Stöffler (1452-1531), explained how to build an astrolabe and then how to use it, according to the very fortunate “construction and use” publishing formula which was common during the 16<sup>th</sup> century (Bennett 2011, 698). An instrumental culture – as we have said – fed also by a fervent and enthusiastic entrepreneurial community of inventors and manufacturers, who placed a variety of instruments on the market applicable to a range of practical activities including, for example, navigation, ballistics and architecture.

The close connection between calculation and mathematical practice thus appears wholly evident, expressed through a material culture which created and produced a wide variety of measuring and surveying instruments. This epistemological and factual contiguity between knowing and doing also

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<sup>22</sup> The work, better known by the title *Cosmographia*, was republished in the 16th century in around fortyfive editions, in at least four languages, by around twenty different printing houses (see *Cosmographia: A Close Encounter* 1998).

<sup>23</sup> On Peter Bienewitz, or Bennewitz (1495-1552), who latinised his name into Petrus Apianus, as he is better known, see Van Ortrooy 1963, Kish 1970, Röttel 1995, Kahl 2005, Kern 2010.

<sup>24</sup> Jemme Reinerszoon Frisius (1508-1555). See O'Connor and Robertson 2002.

<sup>25</sup> English astronomer and mathematician Johannes de Sacrobosco (1195 ca.-1256 ca.) is reputed to be the author of a short work on astronomy, *De sphaera mundi* (1230 ca.), which was extraordinarily popular both in its hand-written and printed versions, as an elementary compendium of Ptolemaic doctrine. The simple nature of the concepts described greatly facilitated its use as a teaching text until almost the late 17<sup>th</sup> century; its popularity increased not only due to the many comments on the work but also due to the introduction, from the first printed version dated 1538 (Wittenberg, Josef Klug), of three volvelles which facilitated the complex calculations that the Ptolemaic cosmological theory required to determine the movement of the planets and their orbits (Gingerich 1994).

<sup>26</sup> We should remember that the University of Vienna was the *alma mater* of scholars of the calibre of Georg von Peuerbach (1423-1461) and Regiomontanus. See Aschbach 1877.

explains the professional contiguity between mathematicians, instrument manufacturers and printers, which often coincided in the same person: just think of Regiomontanus, and then Gemma Frisius and even Apianus, who ran their own technical workshops and printing houses.

In the case of Apianus and his *Cosmographicus liber*, the recipients of the work, which the author refers to as *tyrunculi geographiae*, could in social and cultural terms be identified with a wide range of readers, who were substantially mathematical enthusiasts and amateurs, in the broadest sense of the term “mathematical”, as we have seen, had in the early 16<sup>th</sup> century (Crupi 2018).<sup>27</sup> A wide range of readers therefore who, with the discovery of the New World and the recent circumnavigation of the globe by Magellan (1519-1522), here found the satisfaction of the many curiosities aroused by the journeys of discovery new lands and new peoples and the desire to become intellectually, if not physically, protagonists in the exploration of regions of the world that were still unknown. The rich illustrative apparatus and instruments of the *Cosmographicus liber* were therefore most suited for the private exploration of the new cosmographic world.

Four movable devices were present in the 1524 edition,<sup>28</sup> accompanied by explanatory texts, articulated into propositions illustrating their operation. Of particular interest is the second instrument (Fig. 6), consisting of two threads, a rectangle representing the horizon, a rotating triangle measuring the altitude of the sun and a volvelle indicating the declination of the sun, hours and height of the celestial pole.

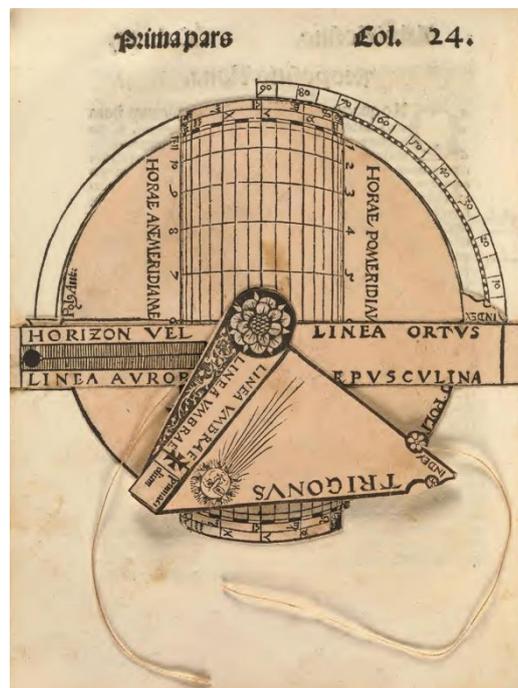


Figure 6. P. Apianus, *Cosmographicus liber* (Antwerp 1533)

<sup>27</sup> For a summary of the historiographical debate over the distinction between “mathematical amateurs” and “mathematical practitioners”, see not only the essay by Vanden Broecke 2000 (in particular, p. 132), but also the stimulating pages by Gaida 2016.

<sup>28</sup> The subsequent editions, both in Latin and in other languages, vary by the number of images and correct, or integrate the original text.

To be correctly activated, this device required the book to be positioned vertically, the lower part pointing towards the sun; this transformed the book itself into a surveying instrument, a movable device identical to those it represented and described. Equally significant, from our point of view, is the *instrumentum syderale* (Fig. 7), used to determine the time of night according to the appearance and position of the last two stars of the Plough in relation to the Pole star.



Figure 7. P. Apianus, *Instrumentum syderale* in *Cosmographicus liber* (Antwerp 1533)

Indeed, Apianus not only depicted the image of the *instrumentum*, but also reproduced the individual components of a “nocturnal” to be cut out and assembled as illustrated<sup>29</sup> (Fig. 8).

<sup>29</sup> For a more accurate description of Apianus’s devices, see: Strano 2019, 111.



Figure 8. P. Apianus, Instrumentum syderale in *Cosmographicus liber* (Antwerp 1533)

A few years later, Apianus printed the marvellous *Astronomicum Caesareum*,<sup>30</sup> one of the most beautiful and spectacular books of the Renaissance period, which he dedicated to Charles V and which earned him not only titles and prebends, but also the trust of the Emperor, who often consulted him on astronomical matters. The book, also intended for amateur astronomers, proposed the Ptolemaic-based astronomical doctrines once more, along with notions of judicial astrology and calendar calculating.<sup>31</sup> The volume, enriched with a wide variety of illustrations, is accompanied by around twenty hand-coloured volvelles, assembled directly at the printing house which Apianus had installed in Ingolstadt (Fig. 9).

<sup>30</sup> The magnificence of the book can be appreciated in the copy available [on-line](#) at the Deutsche Digitale Bibliothek (DDB).

<sup>31</sup> For an accurate description of the astronomical instruments present in the work, see Gislén 2016, Gislén 2017a, Gislén 2017b.



Figure 9. P. Apianus, *Astronomicum Caesareum* (Ingolstadt 1540)

As explained, after Ramon Lull and Matthew Paris, the volvelle was used until the 17<sup>th</sup> century and beyond in a number of entertainment, scientific and educational applications and, however, among the few works intended to compete with the *Astronomicum Caesareum*, in purely aesthetic and paper engineering terms, by far the best are *Dess Menschen Circke*<sup>32</sup> by the eccentric Swiss-German physician and alchemist Leonhard Thurneysser zum Zhurn (1531-1596)<sup>33</sup> and *Astrologia seu Motus, et loca siderum* by Neapolitan cartographer Ottavio Pisani (1575-post 1637).

<sup>32</sup> See the digital edition of the example kept by the Staatsbibliothek in Berlin, made available [online](#). Schirmacher 2016 offers an accurate description of this.

<sup>33</sup> The most accurate biographical contribution on L. Thurneysser is by Yves Schumacher (Schumacher 2011).

The first is a sophisticated paper astrolabe which is part of a series of eight and which is a complement of the second edition of his more complex philosophical work entitled *Archidoxa*<sup>34</sup> (1575), printed almost certainly in his own printing house and perhaps with the contribution of the famous artist and engraver Jost Amman. Composed of rotating discs representing the constellations and other elements of the skies, which according to his own beliefs and those of the period, determined the path and influence of the planets, this very particular astrolabe was designed to allow readers to calculate their own personal horoscope (Fig. 10).



Fig. 10 L. Thurneysser, *Dess Menschen Circkel* (Berlin 1575). Courtesy Staatsbibliothek zu Berlin – Preußischer Kulturbesitz SBB-PK, 2° Libri impr. Rari 641. SBB©Schirmacher

«Gli elementi di ogni astrolabio comprendono una tavola di base riferita al singolo pianeta con i suoi diagrammi astronomici, a questa vanno sovrapposte le due griglie dedicate ai due planisferi celesti, il disco dell'Albero della vita, *Baum des Lebens*, la rondella del Pianeta, la figura del Dragone che rappresenta la linea dei nodi, e altri 3 o 4 rondelle o indicatori con indici che hanno lo scopo di collegare tra loro i dati riportati sui circoli graduati posti sui bordi o all'interno della tavola base» (Stoppa 2016).

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<sup>34</sup> Thurneysser 1575.

The number of elements to cut out, their laced decoration, rich with symbolic figures, and the three-dimensional effect of the device make *Dess Menschen Circkel* a rare and precious object that anticipates, one might say, the virtuosisms of the Baroque culture (Fig. 11 and 12).



Figure 11. L. Thurneysser, *Dess Menschen Circkel* (Berlin 1575). Courtesy Staatsbibliothek zu Berlin – Preußischer Kulturbesitz SBB-PK, 2° Libri impr. Rari 641. SBB©Schirmacher



Figure 12. L. Thurneysser, *Dess Menschen Circkel* (Berlin 1575). Courtesy Staatsbibliothek zu Berlin – Preußischer Kulturbesitz SBB-PK, 2° Libri impr. Rari 641. DHM©Brand

The other work mentioned above, *Astrologia*<sup>35</sup> by Ottavio Pisani, again inspired by the Ptolemaic system, set out as an updated summa of the astronomical knowledge of the time: it is «un'opera spettacolare per il formato atlantico e, soprattutto, per l'elevato numero di complesse tavole mobili colorate a mano che illustrano la struttura della sfera celeste e i moti dei pianeti»<sup>36</sup> (Fig. 13). The book, dedicated to Cosimo II de' Medici, Grand Duke of Tuscany, paying homage to his dynasty, showing the orbits of the de Medici planets in a plate, cost the author two hundred scudos and ten years of work.<sup>37</sup> The volume shows not only Pisani's scientific skills but also an extraordinary technical mastery in the production of planispheres, complex overlapping volvelles and a complex system of jointed hands.

<sup>35</sup> Do not be fooled by the title, the work is in fact about astronomy.

<sup>36</sup> Galluzzi 2014. A digital reproduction of *Astrologia* can be admired on the website of the Galileo Museum in Florence: <https://mostre.sba.unifi.it/tesori-inesplorati/it/98/astrologia-seu-motus-et-loca-siderum>.

<sup>37</sup> Pisani, Ottavio, *Lettera a Galileo Galilei*, 7 November 1613, in G. Galilei, *Le opere*, vol. 11, Florence, Barbera, 1966, p. 592.

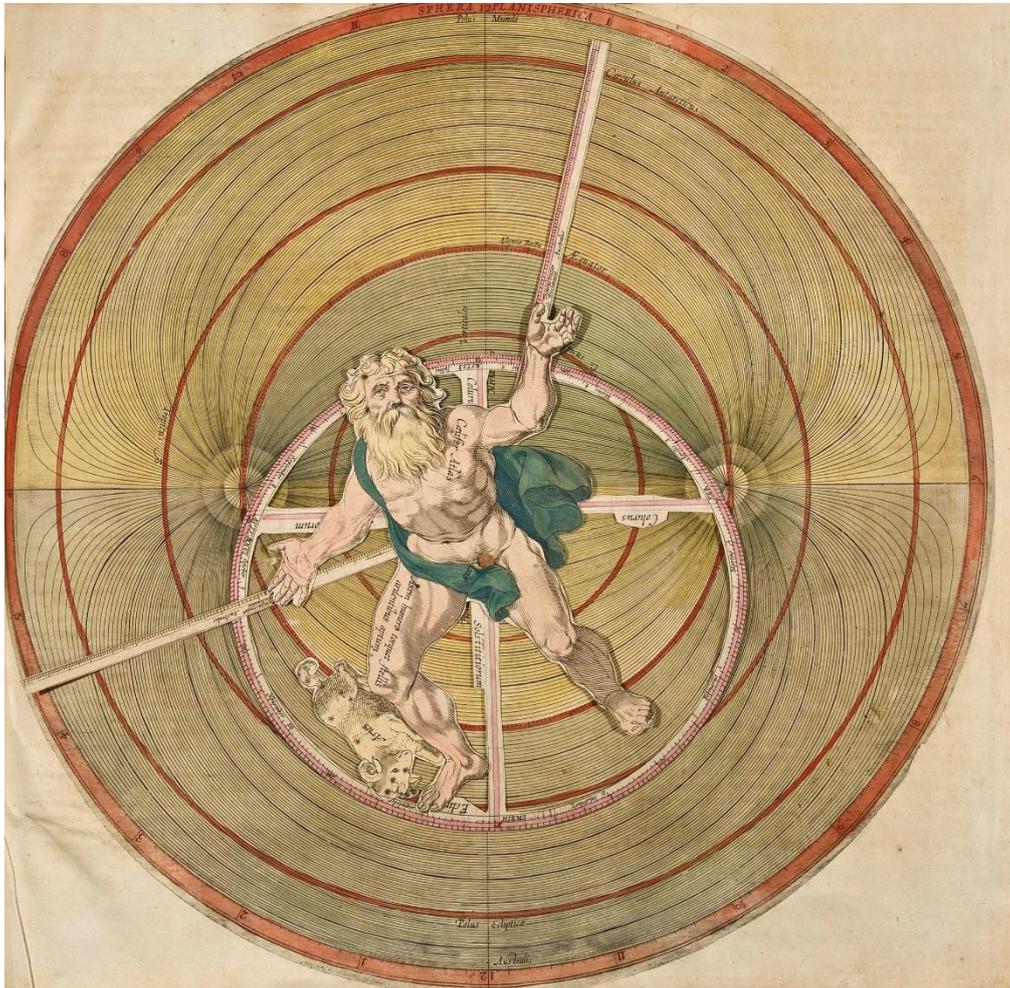


Figure 13. O. Pisani, *Astrologia* (Antwerp, 1613). Courtesy Museo Galileo, Firenze

The volvelle device was also exploited in the field of medical astrology, where predictive manuscripts were common, also in a “da bisaccia” format,<sup>38</sup> used to make astrological forecasts on the basis of the conjunction of the stars, solar and lunar eclipses, the position of the planets and other astronomical variables.<sup>39</sup>

An extraordinary example is the rare collection of four engravings on copper, *The four seasons of human life*, which represent human anatomy, throughout its different ages and, symbolically, through the seasons of nature<sup>40</sup> (Horstmanshoff 2002). Their complexity is characterised not only by their

<sup>38</sup> The definition is by Armando Petrucci 1979, 142.

<sup>39</sup> See Bober 1948, in particular: 23–24; Robbins 1970, in particular: 396–397.

<sup>40</sup> The date of the work is uncertain, although from the 17<sup>th</sup> century (on the basis of some historical and material evidence), and the identity of its author, imagined to be the English physician and alchemist Robert Fludd (1574-1637) or the landgrave from Kassel, Moritz von Hessen-Kassel (1572-1632). The only known example that still exists today is kept at the Duke University Medical Center Library in Durham (USA).

sophisticated symbolism but also by the presence of flaps and volvelles: the first, the result of an authentic magic trick of paper engineering, aim to explore the human body through volvelles a series of anatomical layers; the second, as in the case of the plate *Autumnus*, to calculate the progression of a disease or the calendar of a pregnancy.<sup>41</sup>

## Conclusions

The catalogue of movable books with certainly does not end with the examples we have illustrated here, and it would perhaps be worth attempting a bibliographical enterprise in this sense to reconstruct the map of the whole European production between the 14<sup>th</sup> and 18<sup>th</sup> centuries. This enterprise would allow us to analyse and compare the different types and production characteristics of these works, the socio-cultural composition of their recipients, their methods of use and, last but not least, the economic dimension of a phenomenon, with an in-depth study of the cost of their production and sales. As we said, the epistemological and factual contiguity of knowing and doing also explains the professional contiguity among creators, instrument manufacturers and printers. The phenomenon is significant, if we think of the close economic and cultural connection with scientific, academic and university centres, and with the self-learning practices promoted significantly by the printed book industry. These “enriched books” of the early modern age are hybrid objects that change the cognitive systems of learning, as, at least in their intentions they optimise and speed up some logic operations and calculations. Understanding their use therefore means broadening the horizon of understanding our modernity.

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<sup>41</sup> An effective interactive representation of the work can be found on the following web page:  
<https://library.duke.edu/media/flash/fourseasons/>.

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